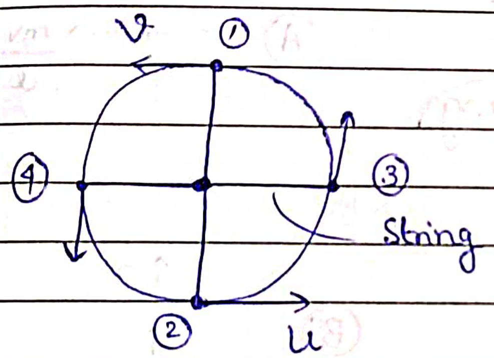
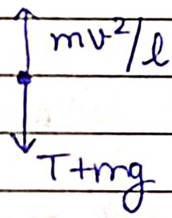
Work, Power & Energy

Vertical Circular Motion -

C1: If body just completeing circle.



At highest pt. in obj's frame,



$$\left(\frac{v^2}{l}\right) = \left(\frac{T}{m}\right) + g$$

$$\Rightarrow \boxed{V_{min} = \sqrt{gl}}$$

vel at highest pt.

By Energy Conservation,

$$E_{①} = E_{②} \Rightarrow \frac{1}{2}mv^2 + mgl = \frac{1}{2}mu^2 - mgl$$

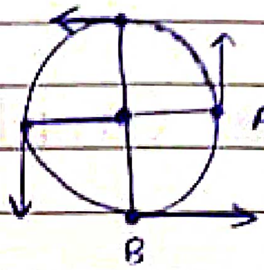
$$\Rightarrow \boxed{u_{req} = \sqrt{5gl}}$$

For completeing circle, $\boxed{u \geq \sqrt{5gl}}$

Date:

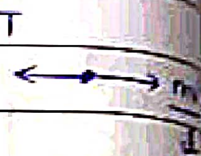
Similarly at ③ & ④, $\boxed{(\text{Vel.}) = v = \sqrt{3gl}}$

Q) Find tension at A & B if obj just complete circle.



A) We know, $v_A = \sqrt{3gl}$

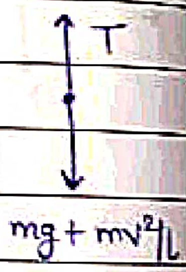
Ⓐ



$$T = \frac{m(3gl)}{l} \Rightarrow T = 3mg$$

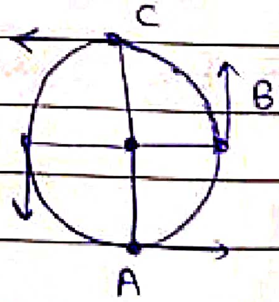
We know, $v_B = \sqrt{5gl}$

Ⓑ

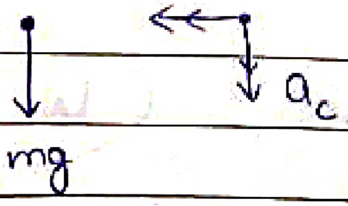


$$T = mg + \frac{mv^2}{l} \Rightarrow T = 6mg$$

Q) Find net acc. of obj at A, B & C if obj just complete circle

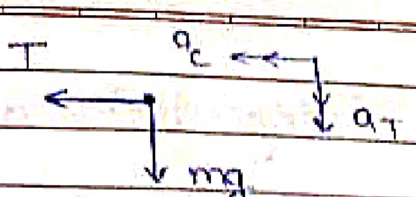


A) At C,



$$a_T = 0$$
$$a_c = \frac{(gl)}{l} \Rightarrow a_{net} = g$$

At B,

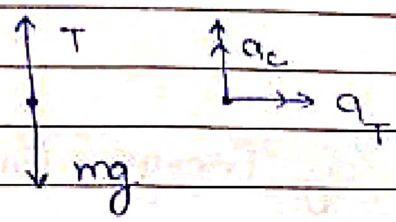


$a_c = \frac{(3gl)}{l}$

$a_c = 3g$ $\Rightarrow$ $a_{net} = g\sqrt{10}$

$a_T = g$

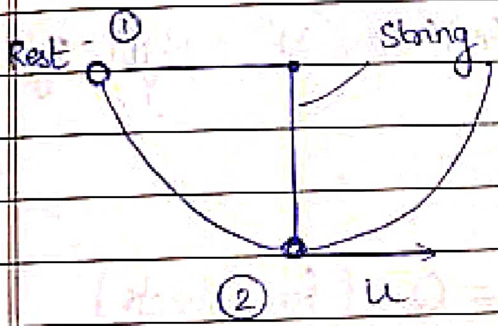
At A,



$a_T = 0$ $\Rightarrow$ $a_{net} = 5g$

$a_c = \frac{(5gl)}{l}$

C2: If Body released from rest horizontally.



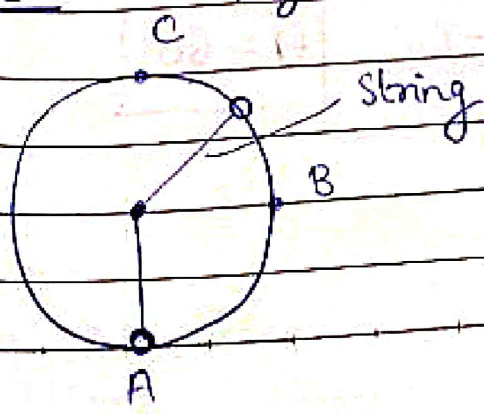
By Energy Conservation,

$E_{①} = E_{②}$

$\Rightarrow mgl = \frac{1}{2}mu^2 \Rightarrow u = \sqrt{2gl}$

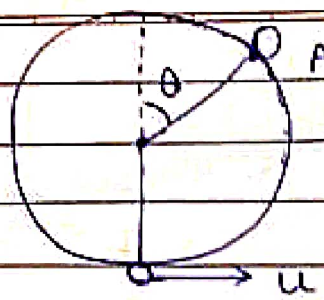
For oscillation, $\boxed{u \leq \sqrt{2gl}}$

C3: $\sqrt{2gl} < u < \sqrt{5gl}$



String will slack at some pt. b/w B & C.

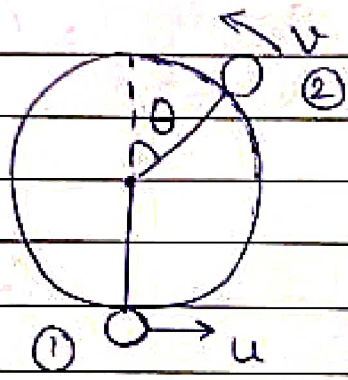
Q)



String slacks at A.

If $u = \sqrt{3.5gl}$, then find θ.

A)



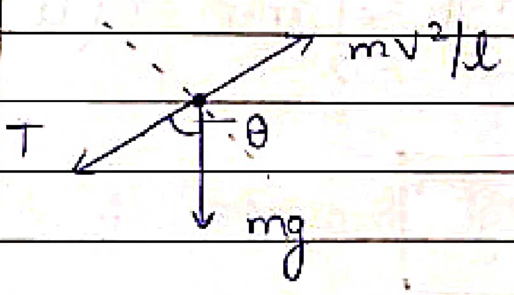
By Energy Conservation,

$$E_{①} = E_{②}$$

$$\Rightarrow \frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgl(1 + c_\theta)$$

$$\Rightarrow v^2 = \left(\frac{7gl}{2}\right) - 2gl(1 + c_\theta) \Rightarrow v^2 = \left(\frac{3gl}{2}\right) - 2glc_\theta$$

In A's frame, $T = 0$ (for slack)



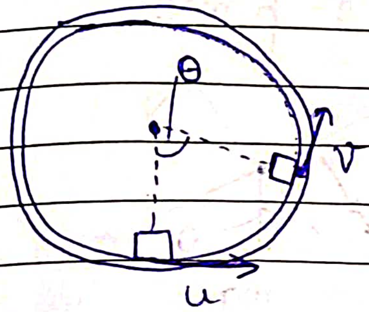
$$T + mgc_\theta = \left(\frac{mv^2}{l}\right)$$

$$\Rightarrow v^2 = glc_\theta$$

Equating, $glc_\theta = \left(\frac{3gl}{2}\right) - 2glc_\theta$

$$\Rightarrow c_\theta = \left(\frac{1}{2}\right) \Rightarrow \boxed{\theta = 60^\circ}$$

C4: If body moving inside vertical circular track.

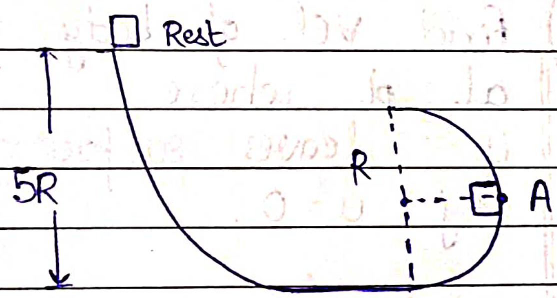


This is similar to earlier cases.

Tension $\longleftrightarrow$ (Normal from Track)

Same results hold.

Q) Find net acc. at A



A) By Energy Conservation, $E_{initial} = E_{final}$

$$\Rightarrow \quad mg(5R) = \frac{1}{2}mv^2 + mgR$$

$$\Rightarrow \quad v^2 = 8Rg$$

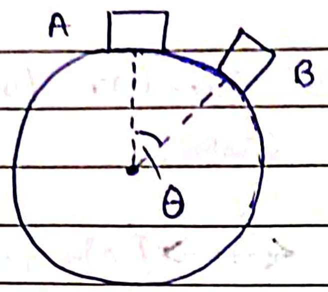
At A,

$a_T = g$

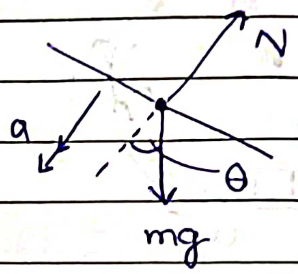
$a_c = 8g$

$$\Rightarrow \quad \boxed{a_{net} = g\sqrt{65}}$$

C5: ~~Motion~~ If body moving outside circle

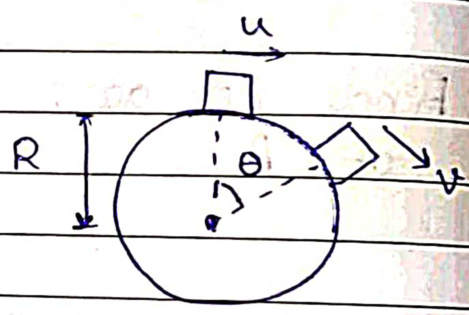


In B's frame,



Body leaves when
$\boxed{N=0}$

Q) Find vel. of body at pt. where it leaves surface.
Obj: $u=0$.

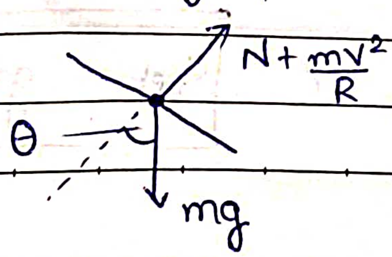


A) By Conservation of Energy,

$E_{initial} = E_{final} \Rightarrow mgR = \frac{1}{2}mv^2 + mgR\cos\theta$

$\Rightarrow v^2 = (2gR)(1-\cos\theta)$

At leaving pt.,



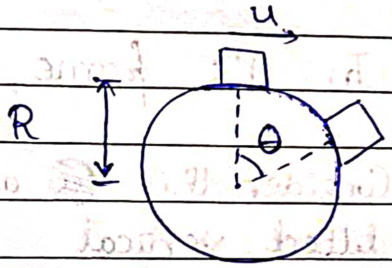
$N=0 \Rightarrow \left(\frac{mv^2}{R}\right) = mg\cos\theta$

$\Rightarrow v^2 = gR\cos\theta$

Equating, $(2gR)(1-c_\theta) = gR\,c_\theta \Rightarrow c_\theta = 2/3$

$$\Rightarrow \boxed{v = \sqrt{2Rg/3}}$$

Q) Find θ at which obj. leaves surface



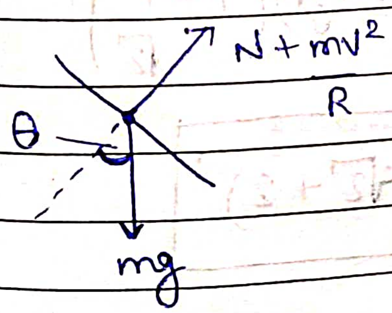
Intial obj. vel: $u = \sqrt{gR/2}$.

A) By Conservation of Energy,

$E_{initial} = E_{final} \Rightarrow \frac{1}{2}mu^2 + mgR = \frac{1}{2}mv^2 + mgR\,c_\theta$

$\Rightarrow \left(\frac{gR}{2}\right) + 2gR = v^2 + 2gR\,c_\theta \Rightarrow v^2 = \left(\frac{5gR}{2}\right) - 2gR\,c_\theta$

At leaving pt., $N = 0 \Rightarrow \left(\frac{mv^2}{R}\right) = mg\,c_\theta$

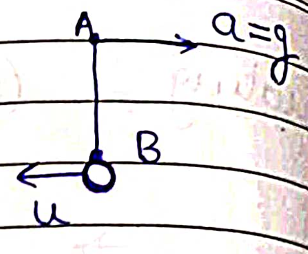


$\Rightarrow v^2 = Rg\,c_\theta$

Equating, $Rg\,c_\theta = \left(\frac{5gR}{2}\right) - 2gR\,c_\theta \Rightarrow \boxed{c_\theta = 5/6}$

Date:

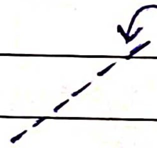
Q) Find min. vel. of mass s.t. it completes full circle.

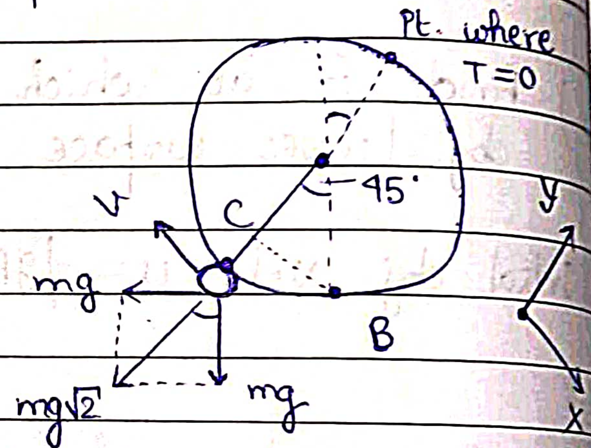


A) Let observer at pt. A

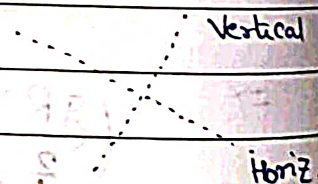
In A's frame,

Consider this as tilted vertical circular motion.





Same results apply $\Rightarrow$ $v_{min} = \sqrt{5(g\sqrt{2})R}$



By Energy Conservation, taking

$E_B = E_C \Rightarrow \frac{1}{2} m u^2 = \frac{1}{2} m v^2 + m g_{eff.} R\left(1 - 1/\sqrt{2}\right)$

$\Rightarrow u^2 = 5(g\sqrt{2})R + (2Rg\sqrt{2})\left(1 - \frac{1}{\sqrt{2}}\right)$

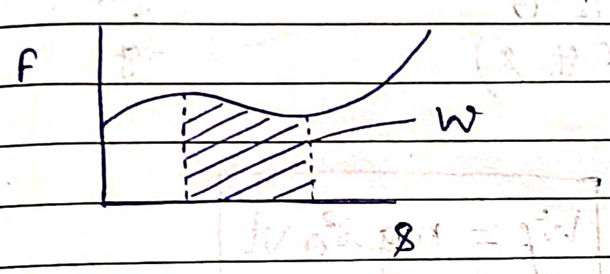
$\Rightarrow \boxed{u = \sqrt{Rg(3\sqrt{2} + 2)}}$

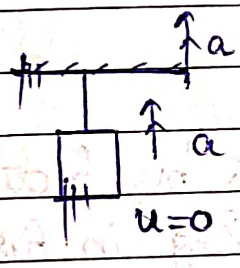
Work

disp. of pt. of application of force

$$\boxed{W = \vec{F}\cdot\vec{s} = Fs\cos(\theta)} \quad \text{if } \vec{F} = \text{Const.}$$

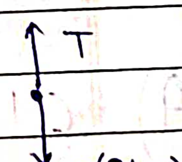
$$\boxed{W = \int \vec{F}\cdot d\vec{s}} \quad \text{if } \vec{F} \neq \text{Const.}$$



Q) 

Find work done by gravity & Tension in first 't' s.
Both ceiling & block moving.

A) In non inertial frame, $T = m(g+a)$



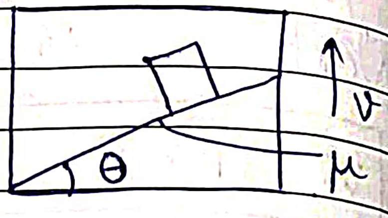
$s = \frac{1}{2}at^2$; $\theta = 0$ (b/w T & s)

$$\Rightarrow W = Fs\cos\theta = m(g+a)\left(\frac{at^2}{2}\right)\cos 0 \Rightarrow \boxed{W_T = \frac{ma(a+g)t^2}{2}}$$

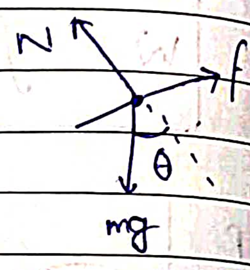
$$\Rightarrow \boxed{W_g = \left(-\frac{ma(a+g)t^2}{2}\right)}$$

Date:

Q) Mass at rest. wrt. lift.
Find work done by friction in first 't' s.



A) In obj's frame, $f = mg\,s_\theta$

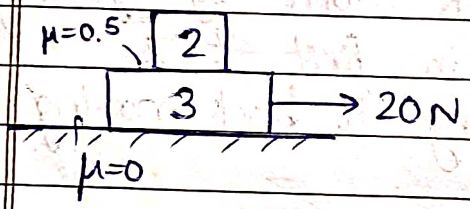


$s = vt$

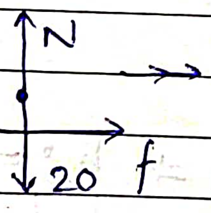
$\angle = \pi/2 - \theta$
(b/w f & s)

$W = f s\, c_\theta$ (Angle) $\Rightarrow$ $\boxed{W_f = mg\, s^2_\theta\, vt}$

Q) Find work by friction on block 2kg in first 2s.



A) C-1 : Combined $a = \left(\frac{20}{5}\right) \Rightarrow a = 4$



$\Rightarrow f = 8$

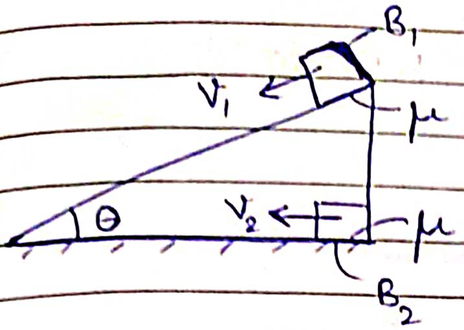
Check: $f_{max.} = (0.5)(20) \Rightarrow f_{max} = 10$

$f_{max.} \geq f \Rightarrow$ Combined motion.

$s = \frac{1}{2}at^2 = (4/2)(2^2) \Rightarrow s = 8$

$W = f s c_\theta \Rightarrow \boxed{W_f = 64}$

Q)

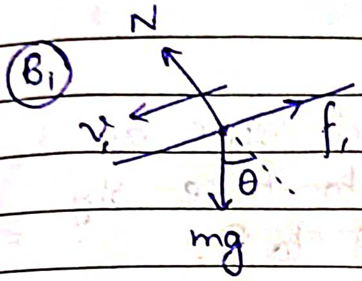


Work by friction on B_1 is W_1

Work by friction on B_2 is W_2

find (W_1/W_2)

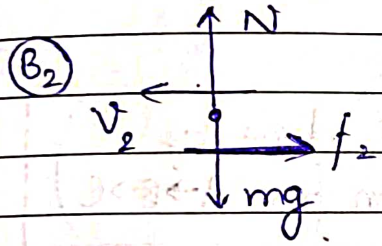
A) B_1



$f_1 = \mu mg c_\theta$, $s_1 = l$

$\Rightarrow W_1 = (-\mu mgl c_\theta)$

B_2

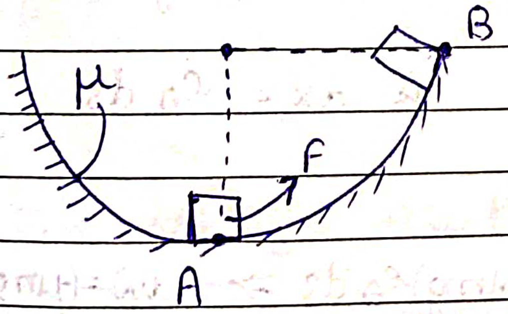


$f_2 = \mu mg$, $s_2 = l c_\theta$

$\Rightarrow W_2 = (-\mu mgl c_\theta)$

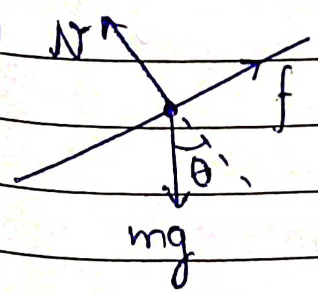
$\Rightarrow \boxed{W_1 = W_2}$

Q)

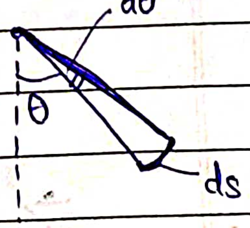


Find work by friction to take block from A to B

A)



$f = \mu mg c_\theta$



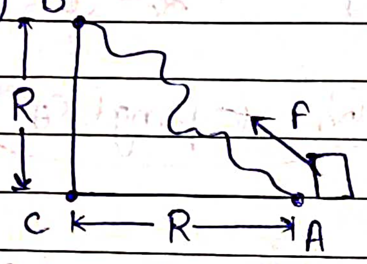
$ds = R d\theta$

Date:

$$dW = \vec{f}\cdot\vec{ds} \Rightarrow \int_0^{W} dW = \int_0^{\pi/2} -\mu mg R\cos\theta \, d\theta$$

$$\Rightarrow \boxed{W = (-\mu mgR)}$$

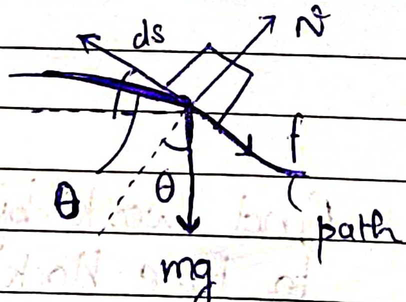
☆ Q)



Find work done by friction to take obj. from A to B.

A) ☆ $\boxed{\begin{pmatrix}\text{Work done by} \\ \text{friction on path}\end{pmatrix} = \begin{pmatrix}\text{Work done by} \\ \text{friction from } A\to B\to C\end{pmatrix}}$

Proof:



$f = \mu mg\cos\theta$

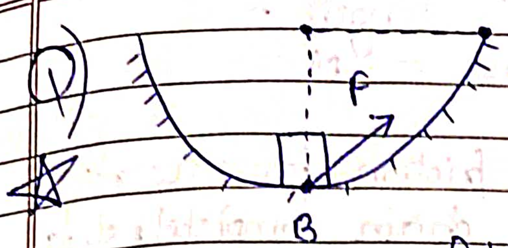
☆ $dx = \cos\theta \, ds$

Now, $dW = f\cdot ds = (-\mu mg)\cos\theta\, ds \Rightarrow dW = (\mu mg)\,dx$

$$\Rightarrow (-\mu mgR) = W$$

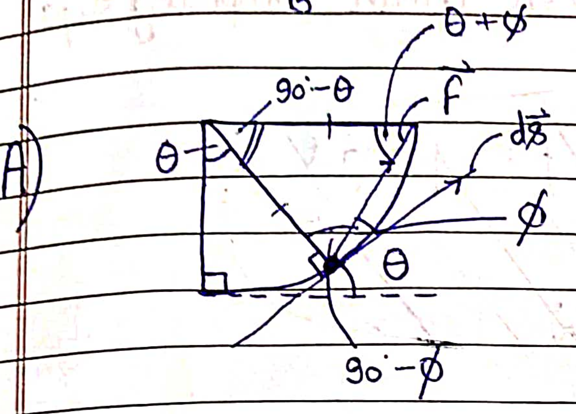
$$\Rightarrow W = (-\mu mgR)$$

Q)



Find work done by force F to take block from A to B, if F always towards A.

A)



In figure,

$$\theta + \phi = 90^\circ - \phi$$

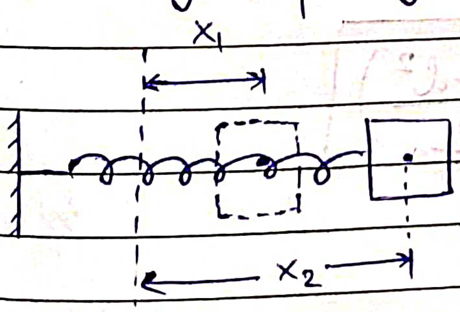
$$\Rightarrow \boxed{\phi = 45^\circ - \theta/2}$$

$$dW = \vec{F}\cdot d\vec{s} \Rightarrow dW = FR\cos(\phi)\,d\theta$$

$$\Rightarrow \int_0^W dW = FR\int_0^{\pi/2} c_{\frac{\pi}{4}-\frac{\theta}{2}}\,d\theta$$

$$\Rightarrow W = FR\left[2\,s_{\frac{\theta}{2}-\frac{\pi}{4}}\right]_0^{\pi/2} \Rightarrow \boxed{W = FR\sqrt{2}}$$

Work by Spring Force



$$dW = \vec{F}\cdot d\vec{x}$$

$$\Rightarrow \int_0^W dW = \int_{x_1}^{x_2} -kx\,dx$$

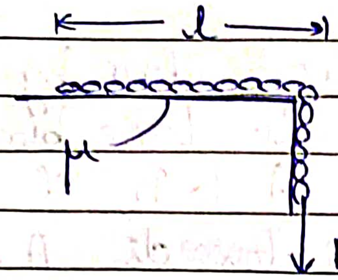
x_2 = final ext. or comp.

x_1 = Init. ext. or comp. $\Rightarrow \boxed{W = (-K/2)(x_2^2 - x_1^2)}$

Date:

Page:

Q)



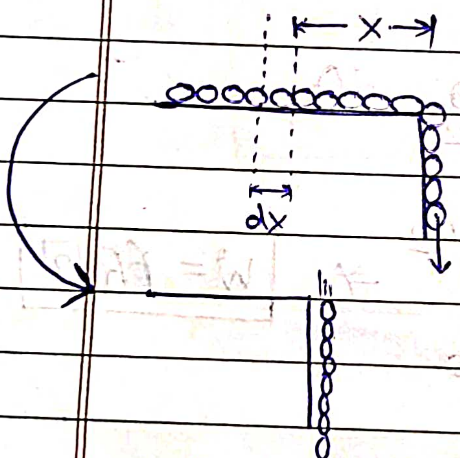
Total length $= L$

Mass $= M$.

Find friction force work when chain completely slips

A)

Consider a portion 'x' dist. away.



This portion travels 'x' m.

$$f = \mu\, dm\, g$$

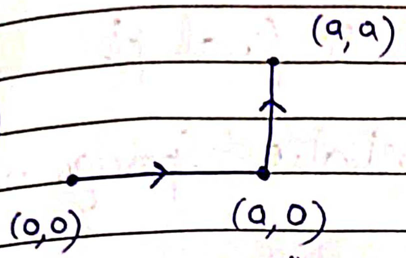
$$\Rightarrow f = \left(\frac{\mu M g}{L}\right) dx$$

Now, $dW = \vec{x}\cdot\vec{f} \Rightarrow \int_0^W dW = \int_0^l \left(-\frac{\mu m g}{L}\right) x\, dx$

$$\Rightarrow \boxed{W = \left(-\frac{\mu m g l^2}{2L}\right)}$$

Q) Let $\vec{F}(x,y) = (-K)(y\hat{i} + x\hat{j})$. find work done in moving body from (0,0) to (a,a) thru (a,0).

A)



$$W_{(0,0)\to(a,0)} = \int F_x\,dx + \int F_y\,dy$$

$$= \int_0^a -K(0)\,dx + \int_0^0 (-Kx)\,dy$$

$$\Rightarrow W_{(0,0)\to(a,0)} = 0$$

$$W_{(a,0)\to(a,a)} = \int F_x\,dx + \int F_y\,dy = \int_a^a (-Ky)\,dx + \int_0^a (-Ka)\,dy$$

$$\Rightarrow W_{(a,0)\to(a,a)} = (-Ka^2)$$

$$\Rightarrow \boxed{\text{Total Work} = (-Ka^2)}$$

Work Energy Theorem

(Total Work done) = (Change in KE)

$$dW = \vec{F}\cdot d\vec{s}$$

$$\Rightarrow dW = m\,a\,ds \Rightarrow \int dW = \int mv\,dv \Rightarrow \boxed{W = \left(\frac{m}{2}\right)\left(v_2^2 - v_1^2\right)}$$

Date:

Conservative Force

Work does NOT depend on path.

Depends only on initial & final pt.

Eg: Gravitational, Electrostatic, Spring, ...

☆ [PE defined ⇒ All forces conservative!]

☆ If force always pt. towards a fix. pt, then force CONSERVATIVE

☆ $$\boxed{\left(\text{Work done by Consrv. forces}\right) = -\left(\text{Change in PE}\right)}$$

Energy Conservation

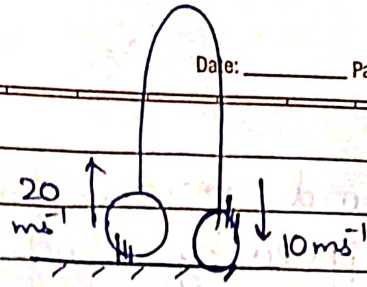
$\Delta KE + \Delta PE = 0$, if $F_{ext} = 0$.

By Work Energy Theorem,

$$W_{Total} = \Delta KE = \underbrace{W_C}_{\text{Consrv.}} + \underbrace{W_{NC}}_{\text{Non Consrv}} + \underbrace{W_{ext}}_{\text{Ext.}}$$

$$\Rightarrow \boxed{W_{ext.} + W_{NC} = \Delta KE + \Delta PE}$$

Q) Find work done by air friction
Mass of obj. 2kg.



A) Obviously, $W_g = 0$. $\Rightarrow$ $W_{air} = \Delta KE$.

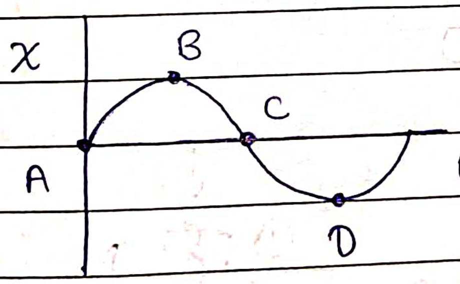
$\Rightarrow W_{air} = \left(\frac{1}{2}\right)(2)(10^2 - 20^2) \Rightarrow \boxed{W_{air} = -300\,J}$

Q) Let $x = t^3 + 2t + 4$ & $m = 2kg$. Find work done by force in 2s.

A) $v = 3t^2 + 2 \Rightarrow v_0^2 = 4,\ v_2^2 = 196$

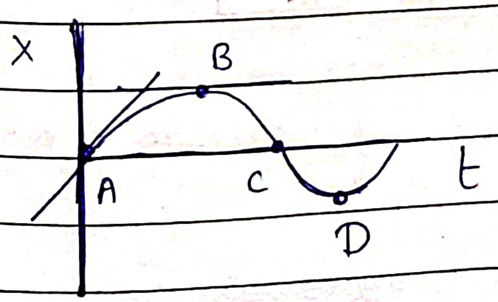
$\Rightarrow W = \Delta KE = \frac{1}{2} \cdot m(v_2^2 - v_0^2) \Rightarrow \boxed{W = 192J}$

Q)



Find work done b/w A & B.
Sign of.

A)



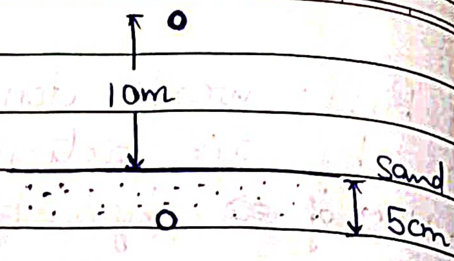
Slope (A) > 0
Slope (B) = 0

$\Rightarrow v_A > 0,\ v_B = 0$

$\Rightarrow W = (\Delta KE) = (1/2)(m)(v_B^2 - v_A^2) < 0 \Rightarrow \boxed{W < 0}$

Date:

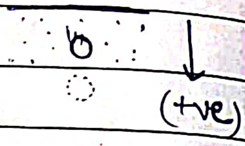
Q) Find resistive force applied by sand.



A) $W_g + W_{sand} = \Delta KE$

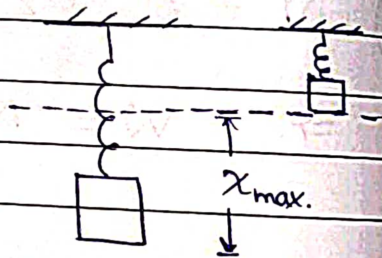
$\Rightarrow$ $\cancel{1kg}$ $(1)(10)(10.05) + (0.05) f_{sand} = 0$

$\Rightarrow$ $\boxed{F_{Avg} = (-2010)\ N}$



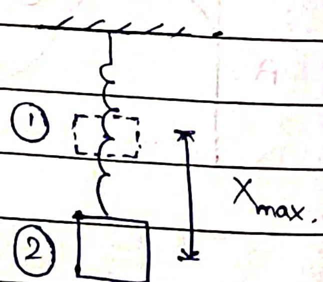
opp. to motion

Q) Find max. extension in spring if obj. released from rest, when spring is relaxed.



A) For x_{max}, $v_{final} = 0$.

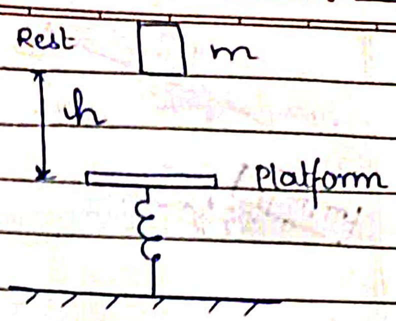
$W_G + W_{spring} = \Delta KE$



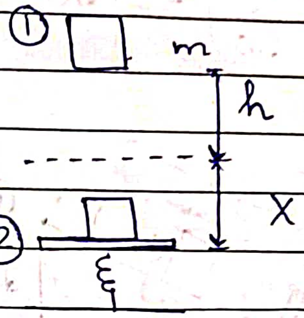
$\Rightarrow$ $mg\, x_{max.} - \frac{1}{2} k x^2_{max.} = 0$

$\Rightarrow$ $\boxed{x_{max.} = \left(\frac{2mg}{k}\right)}$

Q) Obj. released from rest. Find max. compression in spring, if platform massless.



A) For max compression, $v_{final} = 0$.



$E_1 = E_2 \Rightarrow \quad mg(h+x) = \left(\frac{kx^2}{2}\right)$

$(\Delta KE = 0)$

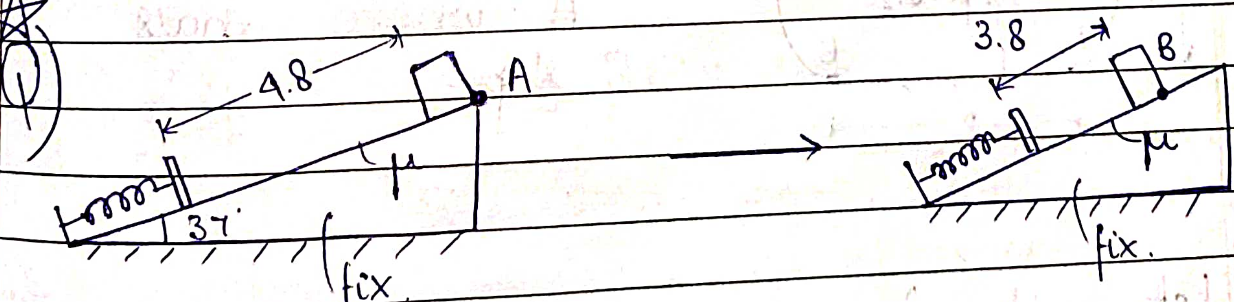
$$\Rightarrow \quad kx^2 - 2mg\,x - 2mgh = 0$$

$$\Rightarrow \quad x = \left(\frac{2mg \pm \sqrt{4(mg)^2 + 8mghk}}{2k}\right)$$

((-ve) value reject as compression req.)

$$\Rightarrow \quad \boxed{x = \left(\frac{mg}{k}\right)\left(1 + \sqrt{mg + 2k}\right)}$$

Q)



Find value of μ & K if max. compression in spring is 0.2.

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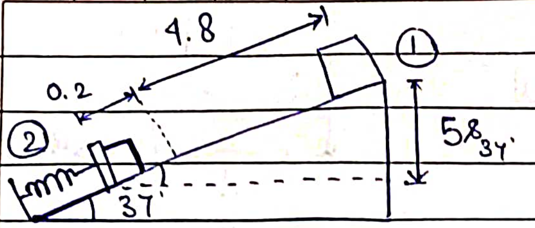
A) 

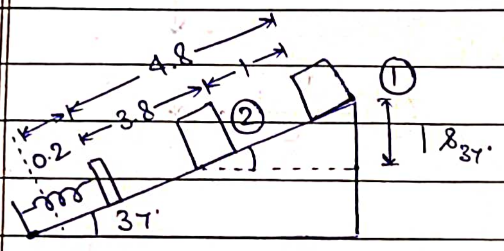
Max. Comp,

$$W_f + W_g + W_s = 0$$

$$\Rightarrow \left(-\mu(1)(10)\left(\frac{4}{5}\right)\right)(5) + (1)(10)(5\cdot 3/5)$$

$$+ \left(-\tfrac{1}{2}k\cdot 4/100\right) = 0$$

$$\Rightarrow \left(\frac{k}{50}\right) = 30 - 40\mu$$



At final,

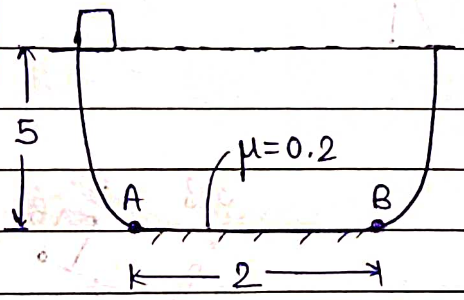
$$W_f + W_g + \underbrace{W_s}_{(0)} = 0$$

$$\Rightarrow (1)(10)(3/5) = \mu\cdot(1)(10)(4/5)\underbrace{(9)}$$

★ Path length

$$\Rightarrow \boxed{\mu = 1/12}$$

$$\Rightarrow \boxed{k = \left(\frac{4000}{3}\right)}$$

★ Q) 

Find dist. from A where mass stops.

A) Let it has covered 'd' dist. on friction surface, before finally ~~finall~~ stopping.

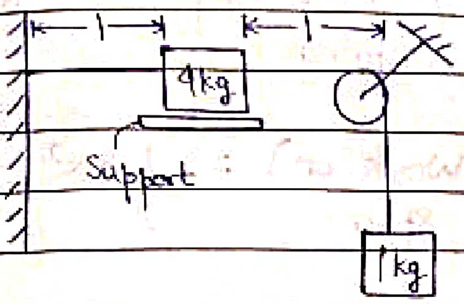
$$W_f + W_g = (\Delta KE) = 0 \Rightarrow (-\mu mg)d + mgh = 0$$

$$\Rightarrow -(0.2)d(mg) + mg(5) = 0 \Rightarrow d = 25$$

In every round it covers 2m, rises up and comes back.

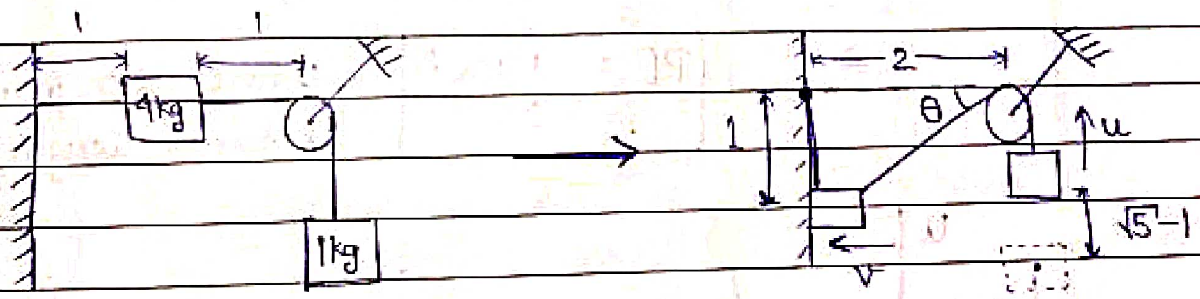
$25 = 2 \cdot 12 + 1 \Rightarrow \boxed{1\,m}$ from A

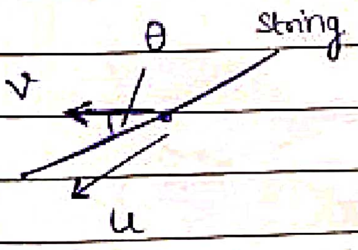
Q) ☆



Find velocity of 4kg when it hits the wall, if support removed.

A)





By String Constraint,

$v\,c_\theta = u \Rightarrow \boxed{u = \left(\frac{2v}{\sqrt5}\right)}$

By Energy Consrv. on System, $W_g = \Delta KE$

$\Rightarrow (4)(10)(1) - (1)(10)(\sqrt5 - 1) = \left(\frac{4}{2}\right)(v^2) + \left(\frac{1}{2}\right)(u^2)$

$\Rightarrow 50 - 10\sqrt5 = (2)(6v^2/5) \Rightarrow \boxed{v = 5\sqrt{\frac{5-\sqrt5}{6}}}$

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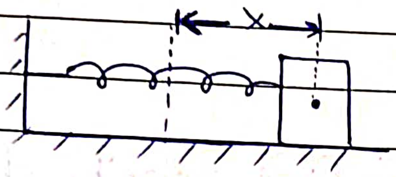
Date:

Energy

Potential Energy –

1) PE stored in a Spring :

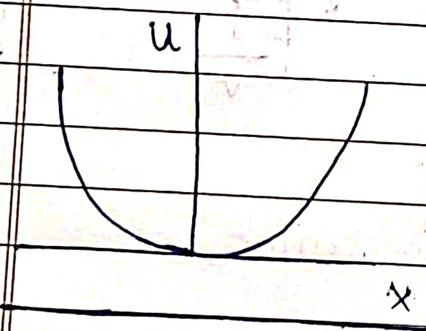


$$\begin{pmatrix}\text{Work by}\\ \text{Spring}\end{pmatrix} = -\frac{1}{2}kx^2$$

$$\Rightarrow \begin{pmatrix}\text{Work on}\\ \text{Spring}\end{pmatrix} = \frac{1}{2}kx^2$$

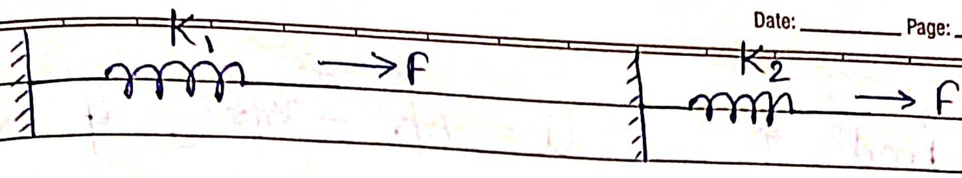
This work done ON spring by mass is stored as PE.

$$\Rightarrow \boxed{PE = \frac{1}{2}kx^2} \quad \begin{pmatrix}\text{when disp. from}\\ \text{natural length}\end{pmatrix}$$



Q) Find U-x graph if ~~work~~ force by hypothetical spring is $f = kx$

A) $W = \int_0^x kx\,dx \Rightarrow W = kx^2/2 \Rightarrow \boxed{U = \left(-\frac{kx^2}{2}\right)}$

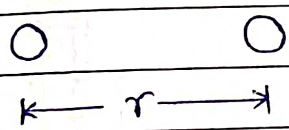
Q) 

Find ratio of ~~work~~ PE stored in both cases.

A) $U_1 = \frac{1}{2}K_1x_1^2 = \frac{1}{2}\frac{(K_1x_1)^2}{K_1} \Rightarrow U_1 = \left(\frac{F^2}{2K_1}\right)$

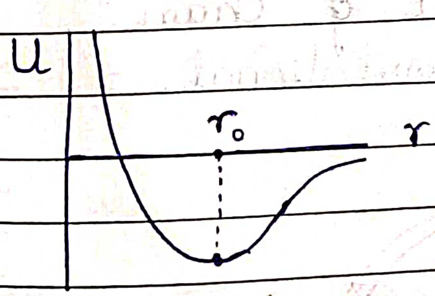
Similarly, $U_2 = \left(\frac{F^2}{2K_2}\right) \Rightarrow \boxed{\left(\frac{U_1}{U_2}\right) = \left(\frac{K_2}{K_1}\right)}$

2) PE b/w molecules of solid :



$$\boxed{U = \frac{A}{r^6} - \frac{B}{r^{12}}}$$

where r = intermolecular dist. ; A, B = const.



☆ $\boxed{F_{(Conserv.)} = \left(-\frac{dU}{dr}\right)}$

r_0 = Equi. Posi. $\Rightarrow$ $U_{min.}$ at $(r = r_0)$

- If $r < r_0 \Rightarrow (dU/dr) < 0 \Rightarrow F > 0 \Rightarrow$ Repulsion
- If $r > r_0 \Rightarrow (dU/dr) > 0 \Rightarrow F < 0 \Rightarrow$ Attraction

Date:

Q) Find r_0 if $U = A/r^6 - B/r^{12}$, if r_0 = Equi Post

A) $F = \left(\frac{-dU}{dr}\right) = -\left(-6Ar^{-7} + 12Br^{-13}\right) = 0$

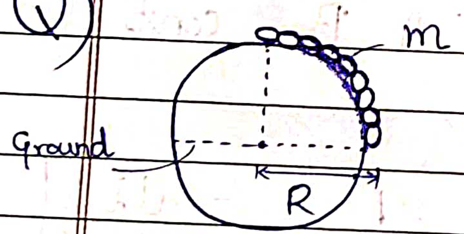
$\Rightarrow \boxed{r = r_0 = \left(\frac{2B}{A}\right)^{1/6}}$

Q) Find U_{min} if $U = A/r^6 - B/r^{12}$

A) U_{min} at $r = r_0 \Rightarrow U_{min} = \frac{A}{(2B/A)} - \frac{B}{(2B/A)^2}$

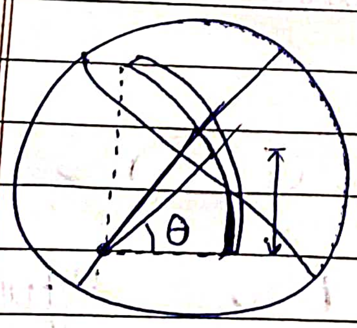
$\Rightarrow \boxed{U_{min} = \left(\frac{A^2}{4B}\right)}$

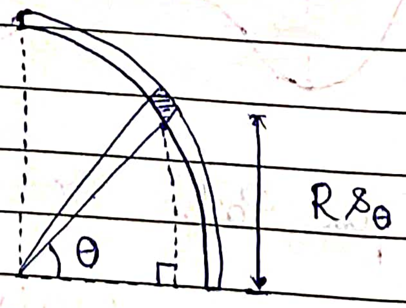
Q)



Find PE of chain. (gravitational.

A)



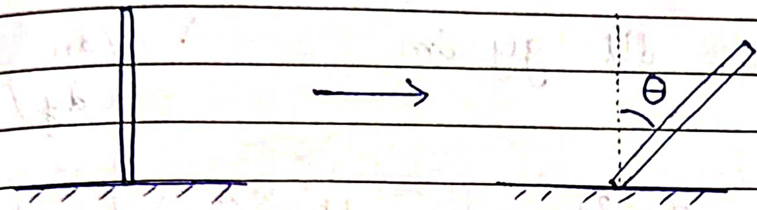


$dU = gRS_\theta \, dm$
(mgh)

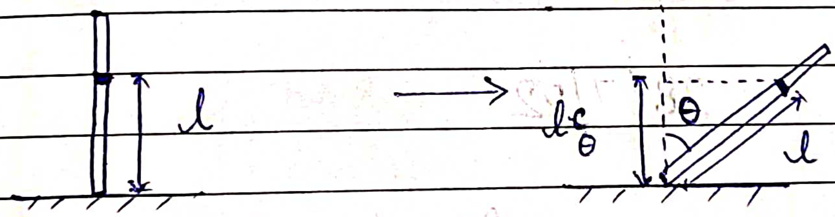
$\rho = \frac{dm}{Rd\theta} = \frac{m}{(\pi R/2)}$

$$\Rightarrow dU = gR s_\theta \cdot \frac{2m}{\pi} d\theta \Rightarrow U = \int_0^{\pi/2} \frac{2mgR}{\pi} s_\theta \, d\theta$$

$$\Rightarrow \boxed{U = \left(\frac{2mgR}{\pi}\right)}$$

Q) 

Find change in PE. if mass of rod 'm' & length of rod 'L'.

A) 

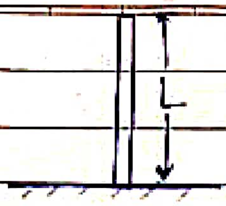
$$dU = g l c_\theta \, dm - g l \, dm \qquad \rho = \left(\frac{dm}{dl}\right) = \left(\frac{m}{L}\right)$$

$$\Rightarrow dU = g l (c_\theta - 1) \cdot \frac{m}{L} dl$$

$$\Rightarrow \Delta U = \left(\frac{mg(c_\theta - 1)}{L}\right) \int_0^L l \, dl \Rightarrow \boxed{\Delta U = \left(\frac{mgL}{2}\right)(c_\theta - 1)}$$

Date:

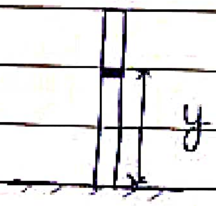
Q)



$\lambda = \left(\frac{\text{mass per unit}}{\text{length}}\right) = 3y$ — dist. from bottom of rod

find gravitational PE

A)



$dU = gy\, dm$ $\lambda = \left(\frac{dm}{dy}\right)$

$\Rightarrow dU = 3gy^2 dy \Rightarrow U = (3g)\int_0^L y^2 dy$

$\Rightarrow \boxed{U = \left(\frac{\text{[illegible]}\, gL^3}{\text{[illegible]}}\right)}$

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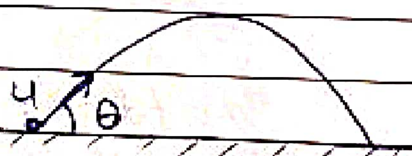
Q)



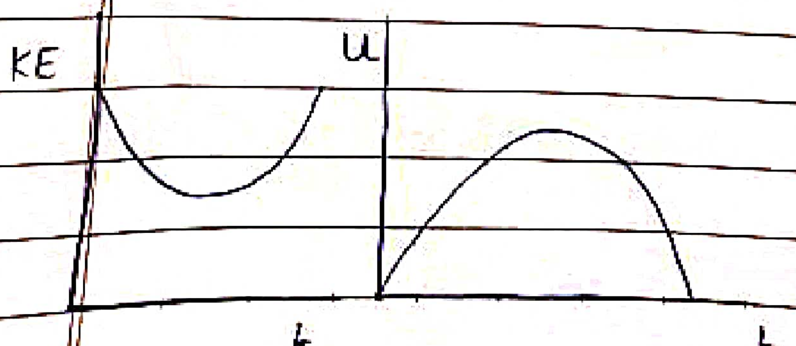
An obj in proj. motion

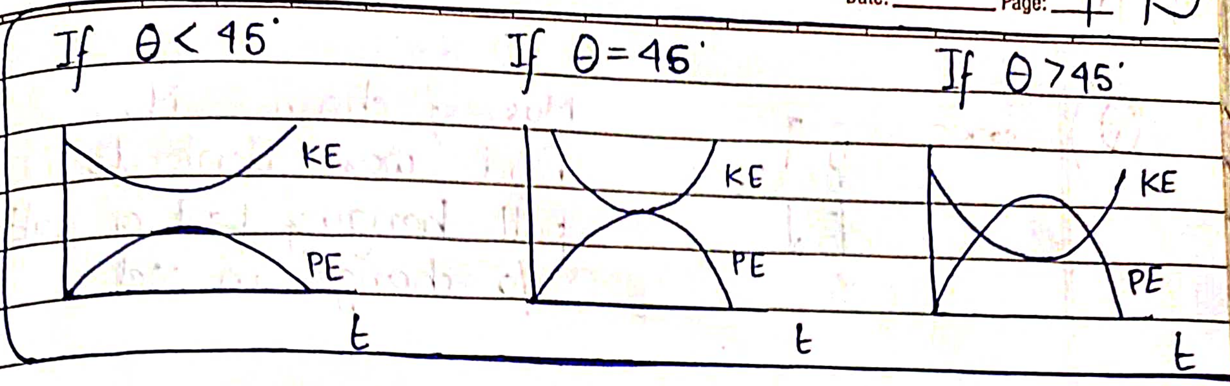
Draw KE-t & U-t graph

A) $U = mgh = \cancel{\left(\frac{mg^2t^2}{2}\right)}$ $(mg)(u_y t - g/_2 t^2)$ $\Rightarrow \boxed{U \propto \text{[illegible]}\left(u_y t - \frac{g}{2}t^2\right)}$

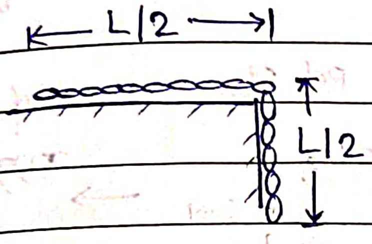
$\vec{v} = \langle u_x, u_y - gt\rangle \Rightarrow KE = \left(\frac{m}{2}\right)\left(u_x^2 + (u_y - gt)^2\right)$

$\Rightarrow \boxed{KE \propto u_x^2 + (u_y - gt)^2}$



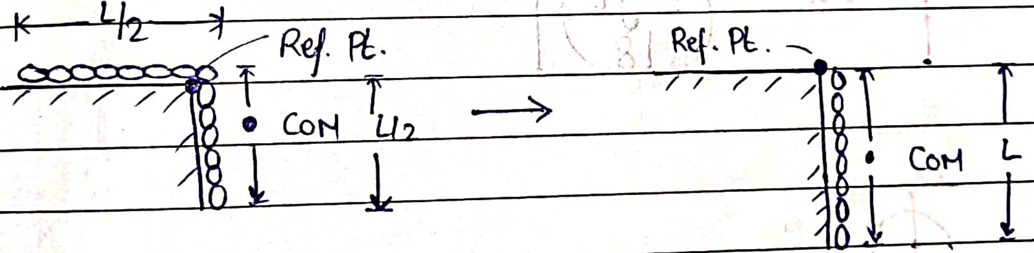


Q)



Initial vel. of chain = 0
Find vel. when chain just falls off.

A) Since no ext. force, apply Energy Consrv.



By Energy Consrv., $\Delta KE + \Delta U = 0$

$$\Rightarrow \left(\frac{1}{2}mv^2 - 0\right) + \left(-mg\left(\frac{L}{2}\right) - \left(\frac{-m}{2}\right)(g)\left(\frac{L}{4}\right)\right) = 0$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{3mgL}{8} \Rightarrow \boxed{v = \sqrt{\frac{3gL}{2}}}$$

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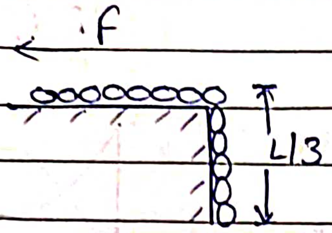
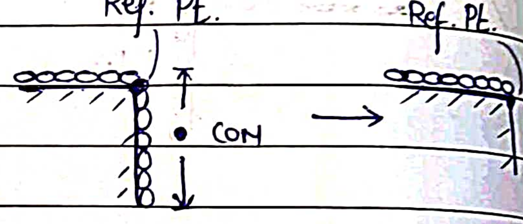
Q) 

Mass of chain = M. Find work done to pull hanging part on table, w/o change in vel.

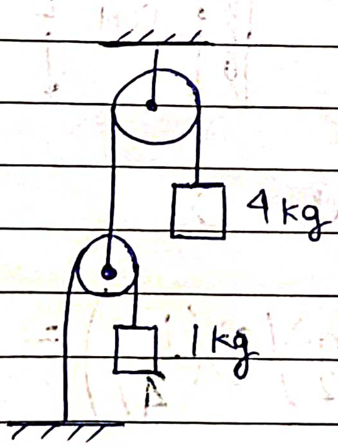
A) $W_{ext} + W_g = \Delta KE = 0 \Rightarrow W_{ext.} = (-W_g)$

$\Rightarrow \boxed{W_{ext.} = \Delta U_g}$

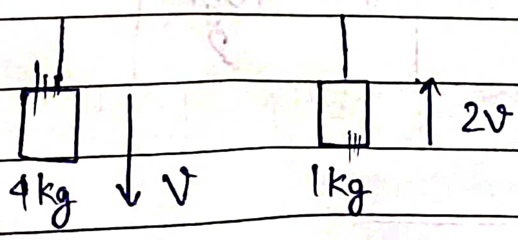


$\Rightarrow W_{ext.} = 0 - \left(\frac{-M}{3}\right) g \left(\frac{L}{6}\right)$

$\Rightarrow \boxed{W_{ext.} = \left(\frac{MgL}{18}\right)}$

Q) 

Find vel. of 1 kg block when 4 kg block moved down by 1 m.

A) 

4 kg goes 1m down

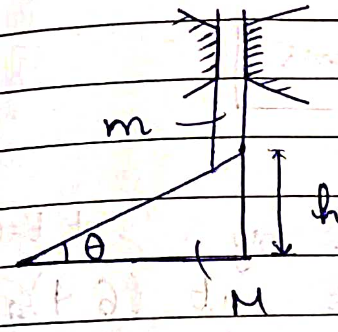
$\Rightarrow$ 1 kg goes 2m up.

By considering block 4 & block 1 as system,

$$\Delta KE + \Delta U = 0 \Rightarrow \left(\frac{1}{2}\right)(1)(2v)^2 + \left(\frac{1}{2}\right)(4)(v)^2 = -(1)g(2) + (4)g(1)$$

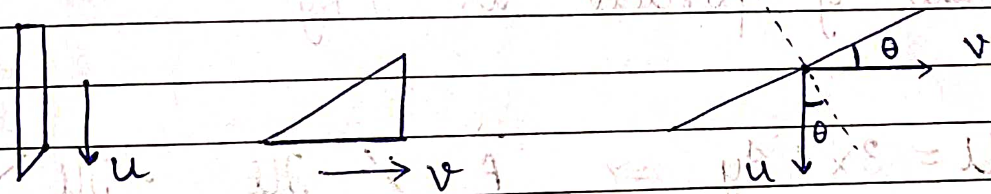
$$\Rightarrow 4v^2 = 2g \Rightarrow 2v = \sqrt{2g} \Rightarrow \boxed{\left(\begin{matrix}\text{Vel.}\\ \text{of 1kg}\end{matrix}\right) = \sqrt{2g}}$$

Q)



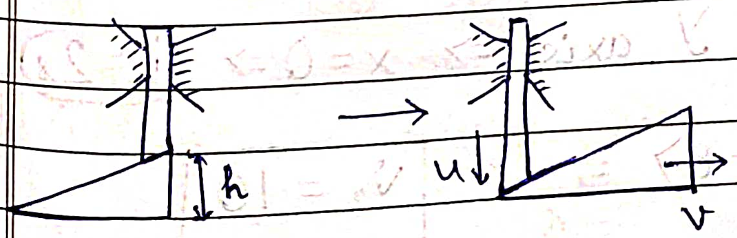
Find vel. of rod when it reaches bottom of incline plane.

A)



By Wedge Constraint, $u c_\theta = v s_\theta \Rightarrow v = u T_\theta$

Considering Rod & Wedge as system,



$$\Delta KE + \Delta U = 0$$

$$\Rightarrow \left(\frac{Mv^2}{2} + \frac{mu^2}{2}\right) = mgh$$

$$\Rightarrow (MT_\theta^2 + m)u^2 = 2mgh \Rightarrow \boxed{u = \sqrt{\frac{2mgh}{MT_\theta^2 + m}}}$$

Q) P.E. of a particle is given by $U = (2 - x^2)$. Particle is at rest at origin. Find vel. of particle at $x = 1$, if mass of particle = 2 kg.

A) $\Delta KE + \Delta U = 0 \Rightarrow \left(\frac{1}{2}\right)(2)(v^2) - 0 = (2 - 0^2) - (2 - 1^2)$

$\Rightarrow v^2 = 1 \Rightarrow \boxed{v = 1\ ms^{-1}}$

Q) PE of a particle is given by $U = 3x + 4y$. It is at rest at $(6, 4)$ at $t=0$. Find velocity as it crosses y axis. Mass of particle is 1 kg.

A) $U = 3x + 4y \Rightarrow \vec{F} = \left\langle -\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y} \right\rangle$

$\Rightarrow \vec{F} = \langle -3, -4 \rangle$

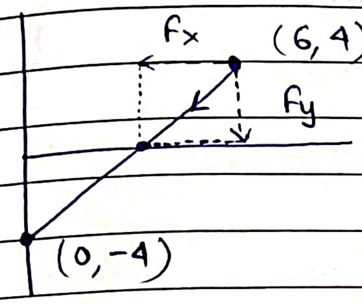
$\Rightarrow \vec{v} = \langle -3t, -4t \rangle \Rightarrow \vec{r} = \left\langle 6 - \frac{3t^2}{2}, 4 - 2t^2 \right\rangle$

When it crosses y axis $\Rightarrow x = 0 \Rightarrow t = 2s$

$\vec{v}(t=2) = \langle -6, -8 \rangle \Rightarrow \boxed{v_2 = 10}$

Better Sol^n : Observe that particle initially at rest and experiences a CONST. force

Motion $\Rightarrow$ Path Strt. Line



$\vec{v}$ always along $\vec{F} = \langle -3, -4 \rangle$

$\Rightarrow$ Particle cut Y axis at $(0, -4)$.

By Energy Consrv.,

$\Delta KE + \Delta U = 0 \Rightarrow \left(\frac{1}{2}\right)(1)(v^2) = 3(6-0) + 4(4+4)$

$\Rightarrow \boxed{v = 10}$

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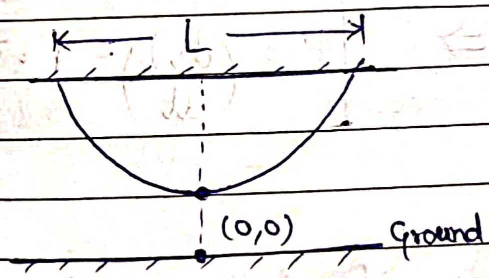
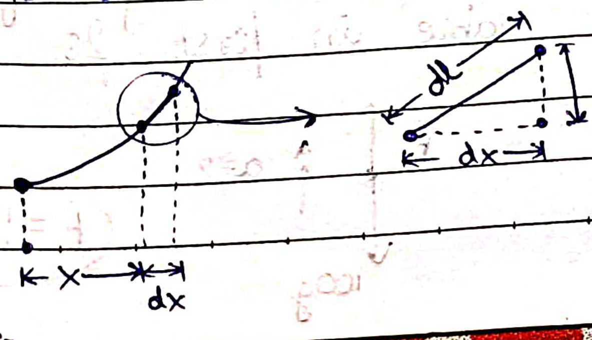
Q) 

Eq^n of chain given is $y = a(e^x + e^{-x})$

If its mass per unit length is λ, find its gravitational P.E.

A) Consider a small chain element dist. 'x' from centre



$$dl = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Now, $dU = gy\,dm = \lambda g y\,dl$

$$\Rightarrow \boxed{dU = (\lambda g)\left(a(e^{x}+e^{-x})\right)\sqrt{1+a^2(e^{x}-e^{-x})^2}}$$

Integrate wrt 'x' to find PE.

Power

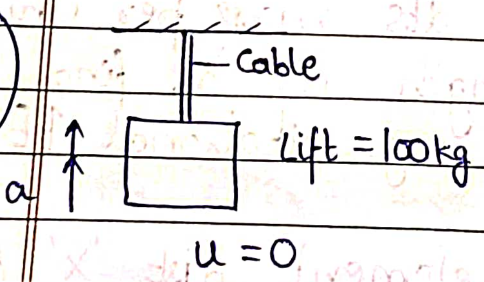
Rate of work done is called power.

$$\boxed{P_{inst.} = \left(\frac{dW}{dt}\right)} \qquad \boxed{P_{Avg.} = \left(\frac{W}{t}\right)}$$

We know, $W = \vec{F}\cdot\vec{s} \Rightarrow \boxed{\left(\frac{dW}{dt}\right)_{inst.} = \vec{F}\cdot\frac{d\vec{s}}{dt} + \frac{d\vec{F}}{dt}\cdot}$

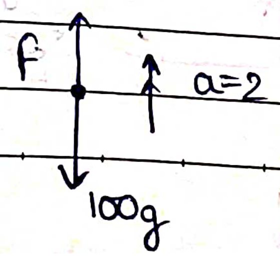
Since ~~If~~ $\vec{F}_{inst.}$ const. wrt time. $\Rightarrow \boxed{P_{inst.} = \left(\frac{dW}{dt}\right) = (\vec{F}\cdot\vec{v})_{inst.}}$

Q)



Mass of lift = 100 kg.
Initial vel. = 0.
Acc. up. = $2\,ms^{-2}$.
Find power developed by cable in first 2s.

A) $P_{Avg} = \left(\frac{W}{t}\right) = \left(\frac{F\cdot s}{t}\right)$



$F = 1200$

In first 2s, $S = \frac{1}{2}at^2 = \frac{2 \cdot 2^2}{2} \Rightarrow s = 4$

$P_{Avg} = \left(\frac{1200 \cdot 4}{2}\right) \Rightarrow \boxed{P_{Avg} = 2400}$

Q) In above Q, find power developed at $t = 2s$.

A) $P = \vec{F} \cdot \vec{v}$ (F = const.) $\Rightarrow P = 1200v$

At $t = 2s$, $v = at = 2 \cdot 2 \Rightarrow v = 4 \longrightarrow \boxed{P = 4800}$

☆ If an obj. starts from rest and moves with const. acc. = a then,

$\boxed{P_{Avg} = \left(\frac{P}{2}\right)}$ at all inst.

Proof: $P_{Avg} = \frac{F \cdot s}{t} = \frac{F}{t} \cdot \frac{1}{2}at^2 = \frac{F}{2} \cdot at = \frac{F}{2} \cdot v$

$\Rightarrow P_{Avg} = P/2$

Q) ☆ A body is given const. power. Find rel^n b/w disp. of body & time, if it start from rest.

A) $P = \left(\frac{dW}{dt}\right) \Rightarrow \int_0^W dW = \int_0^t P\,dt \Rightarrow W = Pt$ (as P const.)

By work energy theorem, $\frac{1}{2}mv^2 = W$

Now, $Pt = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2P}{m}}\, t^{1/2}$

$$\Rightarrow \boxed{s = \left(\frac{2}{3}\right)\sqrt{\frac{2P}{m}}\, t^{3/2}}$$

Better Sol^n : $[P] = [ML^2T^{-3}]$

Now, $ML^2T^{-3} = \text{Const.} \Rightarrow \left(\frac{L^2}{T^3}\right) = \text{Const.}$

$\Rightarrow L \propto T^{3/2}$

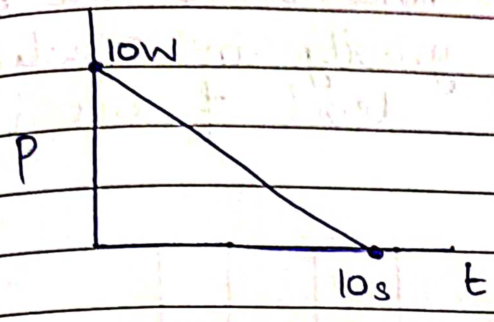
Q) A body is moving in circle of radius R. Centripetal force acting on mass is given by $F_c = mk^2Rt^2$. Find power developed by all forces on the mass

A) Power delivered only by F_T as work done by it $\neq 0$.

Now, $F_c = \left(\frac{mv^2}{R}\right) = mk^2Rt^2 \Rightarrow v = krt$

$F_T = m\left(\frac{dv}{dt}\right) = ma_T \Rightarrow F_T = mkR$

$P = \vec{F} \cdot \vec{v} = F_T v_T \Rightarrow$ $\boxed{P = mk^2R^2t}$

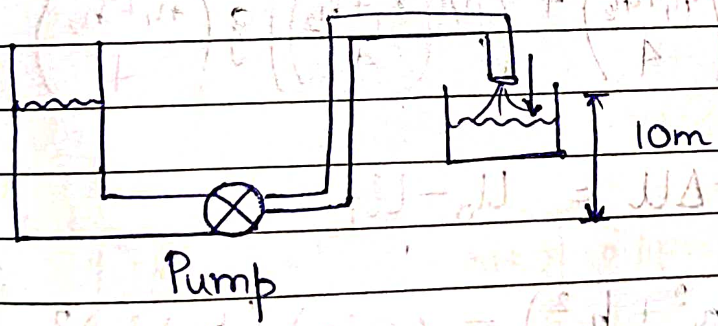
Q) 

If mass of particle = 2kg and at $t=0$, $v=0$; find vel. of particle at $t=10s$.

A) $W = \int P\,dt$ = (Area under P-t curve)

Also by Work Energy Theorem, $W = \frac{1}{2}mv^2$

Equating, $\left(\frac{1}{2}\right)(2)v^2 = \frac{(10)(10)}{2} \Rightarrow \boxed{v = 5\sqrt{2}}$

Q) 

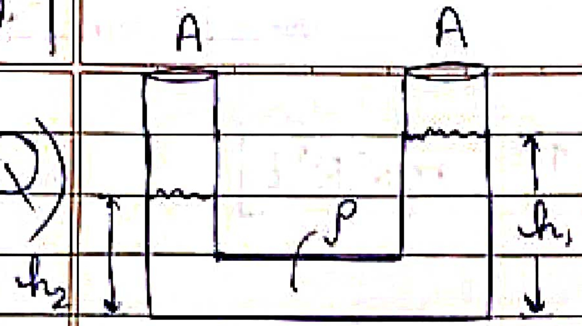
Pump is 80% efficient. Velocity of exiting water is zero. Mass flow rate is $2\ kg\,s^{-1}$
Find power given to pump.

A) $P = \left(\frac{W}{t}\right) = \left(\frac{m}{t}\right)gh \Rightarrow P = \left(2\frac{kg}{s}\right)\left(10\frac{m}{s^2}\right)(10m)$

$\Rightarrow P = 200$ — Power delivered by pump

$P_{delivered} = (80\%)\,P_{given} \Rightarrow \boxed{P_{given} = 250}$

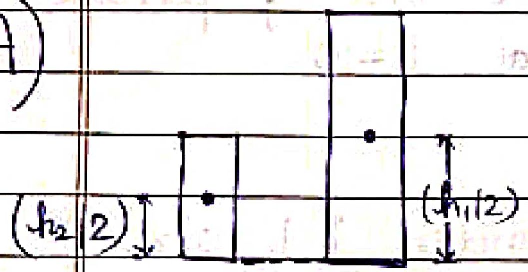
Q) ☆



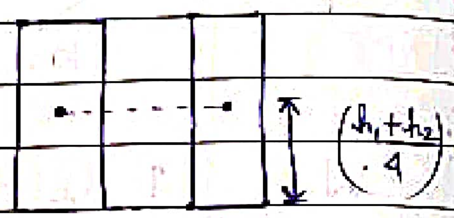
Area of Cross Section $= A$
Density of fluid $= \rho$

Find work done by gravity in equalising level of liquid.

A)



Initial



After

$$U_0 = \left(\rho A \frac{h_2}{2}\right) g \left(\frac{h_2}{2}\right) + \left(\rho A \frac{h_1}{2}\right) g \left(\frac{h_1}{2}\right)$$

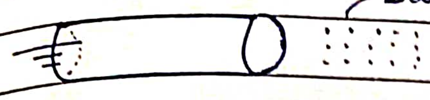
$$U_1 = \left(\rho A \left(\frac{h_1+h_2}{4}\right)\right) g \left(\frac{h_1+h_2}{4}\right) + \left(\rho A \left(\frac{h_1+h_2}{4}\right)\right) g \left(\frac{h_1+h_2}{4}\right)$$

Now, $W_g = -\Delta U = U_0 - U_1$

$$\Rightarrow W_g = \left(\frac{\rho A g}{4}\right)\left(h_1^2 + h_2^2\right) - \left(\frac{\rho A g}{8}\right)\left(h_1+h_2\right)^2$$

$$\Rightarrow \boxed{W_g = \left(\frac{\rho A g}{8}\right)\left(h_1 - h_2\right)^2}$$

Q) Dust particles



Cross Section $= 10^{-2}\, m^2$

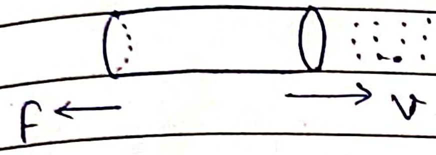
Vel. of Cylinder $= 10^{3}\, ms^{-1}$

Mass $= 10^{-2}$ kg

$\rho_{dust} = 10^{-3}\, kg\, m^{-3}$

Find X coordinate of cylinder at time 't'

A)



$$F = -v\left(\frac{dm}{dt}\right)$$

Now, $\left(\frac{dm}{dl}\right) = \rho A = 10^{-5} \Rightarrow \boxed{\left(\frac{dm}{dt}\right) = 10^{-5}\, v(t)}$

$$\Rightarrow m(t) \cdot a(t) = -v(t) \cdot 10^{-5}\, v(t)$$

$$\Rightarrow \left[10^{-2} + x(t) \cdot 10^{-5}\right] \cdot a(t) = \left(-10^{-5}\right)\left[v(t)\right]^2$$

$$\Rightarrow \left[10^{3} + x(t)\right] \ddot{x}(t) = -\left(\dot{x}(t)\right)^2$$

$$\Rightarrow \int_0^t -\frac{\ddot{x}(t)}{\dot{x}(t)} = \int_0^t \frac{\dot{x}(t)}{10^3 + x(t)} \Rightarrow -\ln\left|\frac{v(t)}{10^3}\right| = \ln\left|\frac{10^3 + x(t)}{10^3}\right|$$

$$\Rightarrow \int_0^t 10^6 = \int_0^t \left[10^3 + x(t)\right] \dot{x}(t) \Rightarrow 10^6 t = 10^3 x(t) + \frac{(x(t))^2}{2}$$

$$\Rightarrow \boxed{\left[x(t)\right]^2 + 2 \cdot 10^3\, x(t) - 2 \cdot 10^6\, t = 0}$$